Faculty of Mathematics, Informatics and Mechanics, the University of Warsaw: topics of the entrance examination into the PhD program in mathematics.

List of elementary level mathematical topics for candidates for doctoral studies in mathematics or applied mathematics.

The list reflects the range of topics obligatory for all candidates. The items on the list have the character of examples. A candidate receives three questions concerning topics presented below and answers two of them chosen by himself/herself.

1. Real and complex numbers and their properties. Sequences and their limits. Bolzano-Weierstrass theorem. Cauchy sequences. Criteria for existence of limit.

2. Series of real and complex numbers. Criteria of series convergence. Absolute and conditional convergence. Multiplication of series.

3. Functions. Continuity and uniform continuity of functions. Properties of continuous functions defined on a compact set. Darboux property.

4. Differential calculus of real functions of a single variable. Rolle and Lagrange theorems. Investigation of behaviour of functions.

5. Series of functions. Pointwise and uniform convergence. Power series. Radius and disk of convergence. Taylor expansion.

6. Indefinite integral. Riemann integral. Improper integrals.

7. Partial derivatives and directional derivative. Gradient. Jacobian. Extrema of functions of several variables. Implicit functions.

8. Measure theory and Lebesgue integral. Passing to the limit under the sign of integral. Fubini theorem.

9. Line and surface integrals. Gauss-Ostrogradski, Green and Stokes theorems.

10. Analytical functions. Cauchy-Riemann equations. Cauchy integral formula. Maximum principle.

11. Banach space. Linear functionals and operators. Dual space. Hilbert space. Lp spaces. Spaces of continuous functions.

12. Determinants and linear equations. Linear and affine spaces. Algebraic sets of degree I and II and their classification.

13. Groups. Cyclic groups. Permutation groups. Group homomorphisms. Kernel. Normal subgroup and quotient group. Lagrange theorem about order of a subgroup.

14. Commutative rings. Ideals. Maximal and prime ideals. Ring homomorphisms. Zero divisors. Invertible elements. Field of fractions.

15. Fields. Prime fields. Characteristic of a field. Algebraically closed fields, fundamental theorem of algebra. Roots of unity.

16. Metric and topological spaces. Methods of introducing topology. Operations on spaces. Tikhonov theorem.

17. Continuous mappings. Tietze theorem.

18. Separable spaces. Connected spaces. Compact spaces.

19. Complete metric spaces. Cantor set and its properties.

20. Fundamental group. Jordan curve theorem. Brouwer fixed point theorem.

21. Conditional expectation, definition, properties, basic characteristics and simple examples for discrete and continuous random variables.

22. Different notions of convergence of sequences of random variables. Laws of large numbers and central limit theorem.

23. Linear differential equations with constant coefficients.

24. Theorems on existence and uniqueness of solutions to ordinary differential equations.

List of extended level mathematical topics for candidates for doctoral studies in mathematics or applied mathematics.

The list reflects the range of topics obligatory for all candidates. The items on the list have the character of examples. A candidate chooses two out of four areas presented below and receives one question out of the two chosen areas.

I Statistics and numerical methods.
Sufficient statistic, definition and properties. Testing of statistical hypotheses; significance level and statistical power. Estimation theory, Cramer-Rao bound. Numerical methods of solving linear and nonlinear equations, numerical correctness. Numerical quadrature rules. Degree and error of a quadrature rule. Numerical approximation of functions.

II Partial differential equations.
Basic types of boundary (boundary-initial value) problems for elliptic, parabolic and hyperbolic equations. Well-posedness of a problem in the sense of Hadamard. Classical and generalized solutions. Lax-Milgram theorem. Maximum principles. Sobolev embeddings and trace theorems. Applications of fixed points theorems to nonlinear partial differential equations. Relationship between elliptic equations and variational problems. Galerkin method. Distributions and fundamental solutions.

III Algebra and topology.
Baire category theorem. Covering spaces, fundamental group, universal covering. Projective spaces. Polyhedra. Topological manifolds. Finite fields. Tensor product. Divisibility in integral domains. Modules over a ring. Algebraic elements over a field. Degree of an algebraic element. Splitting field of a polynomial. Automorphisms of a field. Algebraic sets.

IV Analysis and differential geometry.
Smooth submanifolds in Euclidean space and their maps. Curvature and torsion of a curve in Euclidean 3-space. First and second fundamental form on a manifold. Principal curvatures and directions. Gaussian and mean curvature. Christoffel symbols and Theorema Egregium. Parallel transport. Geodesic curves. Differentiable manifolds, charts and atlases. Riemannian manifolds. Manifolds with boundary. Stokes theorem on manifolds. Conformal mappings. Compact operators on Banach spaces.